An N-atom Collective State Atomic Interferometer with Ultra-High Compton Frequency and Ultra-Short de Broglie Wavelength, with root-N Reduction in Fringe Width

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We describe a collective state atomic interferometer with fringes as a function of phase narrowed by $\sqrt{N}$ compared to a conventional interferometer, N being the number of atoms, without entanglement and violation of the uncertainty limit. This effect arises from the interferences among collective states, and is a manifestation of interference at a Compton frequency of ten nonillion Hz, or a de Broglie wavelength of ten attometer, for $N = 10^6$. The detection process, being a measure of the amplitude of a collective state, can yield a net improvement of phase measurement by as much as a factor of 10.

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Matter wave interferometry has proven to be a potent technology in precision metrology. Atom interferometers have been successfully demonstrated as gyroscopes and accelerometers [1, 2], gravity gradiometers [3, 4], matter-wave clocks [5] and may lead to a more accurate measurement of the fine structure constant than what is currently known [6, 7]. They also form strong test-beds for experiments to measure Newton's gravitational constant [8], gravitational red-shift [9] and for testing universality of free fall [10].

The building block of a Conventional Raman Atom Interferometer (CRAI) is a three level atom in the Λ-configuration, with two metastable states, $|g, p_z = 0\rangle \equiv |g, 0\rangle$ and $|e, p_z = \hbar(k_1 + k_2)\rangle \equiv |e, \hbar k\rangle$ and an excited state $|a, p_z = \hbar k_1\rangle \equiv |a, \hbar k_1\rangle$ coupled by two Raman-resonant counter propagating laser beams, with a single photon detuning δ as shown in Fig. 1(a). One of the laser beams, with Rabi frequency Ω_1, couples $|g, 0\rangle$ to $|a, \hbar k_1\rangle$, while the other laser, with Rabi frequency Ω_2, couples $|a, \hbar k_1\rangle$ to $|e, \hbar k\rangle$. In the limit of $\delta \gg \Omega_1, \Omega_2$, the interaction can be described as an effective two level system excited by an effective traveling wave with a momentum $\hbar k = \hbar(k_1 + k_2)$, with a Rabi frequency $\Omega = \Omega_1\Omega_2/2\delta$ (Fig. 1(b)) [11]. We assume that $\delta \gg \Gamma$, where Γ is the decay rate of $|a\rangle$ so that the effect of Γ can be neglected. On being subjected to a sequence of $\pi/2$-dark-π-dark-$\pi/2$ pulses as illustrated in Fig. 1(c), the atomic wavepacket first separates in to two components owing to the $\hbar k$ momentum transfer from the $\pi/2$ pulse, then gets redirected and finally recombined to produce an interference which is sensitive to any possible phase difference, $\Delta\phi$ between the two paths. The amplitude of $|g\rangle$ at the end of the interferometric sequence varies as $\cos^2(\Delta\phi/2)$. Further theoretical and experimental details of this can be found in ref. 12 and 13.

In the application of a CRAI as a gyroscope, $\Delta\phi$ is the measure of its rotation sensitivity, arising due to the Sagnac effect. The $\Delta\phi$ induced due to rotation at the rate of Ω_G along an axis normal to the area, Θ of the interferometer is given by, $\Delta\phi = 4\pi\Theta m\Omega_G/h$, where m is the mass of the atom [13, 14]. This expression can be derived by two different methods. In the first method, the path difference of the two counter-propagating waves arising due to rotation is multiplied by $2\pi/\lambda_{dB}$, where λ_{dB} is the de Broglie wavelength of the atom, to arrive at the phase difference. The second method follows a more thorough derivation of the phase shift by invoking the relativistic addition of velocities to find the time lag, $\Delta T = 2\Theta\Omega_G/c^2$ in the arrival of the two branches of the wave, where c is the speed of light. $\Delta\phi$ is then the product of ΔT and the wave frequency. In the case of CRAI, this frequency is the Compton frequency of the atom, $\omega_C = \gamma mc^2/\hbar \approx mc^2/\hbar$, where the relativistic time dilation factor, γ, is close to unity for non-relativistic ve-

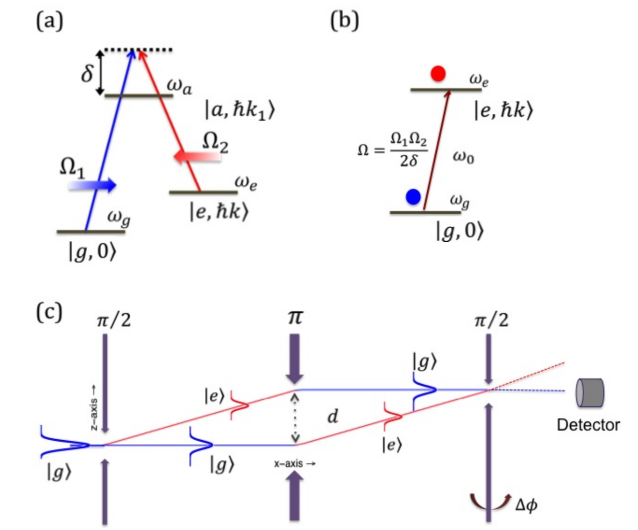

FIG. 1. (a) A three level atom. (b) An equivalent reduced two-level atom model. (c) A single atom interferometer produced via $\pi/2 - \pi - \pi/2$ sequence of excitation.

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locities. As discussed in greater detail in ref. 15, these approaches are equivalent due to the fact that the de Broglie wavelength is merely the laboratory frame manifestation of the Compton frequency induced phase variation in the rest frame of the atom [5, 16–18].

The dependence of $\Delta\phi$ on ω_C (and therefore, λ_{dB}) of the atom has motivated the quest for matter wave interferometry with large molecules. To date, the largest molecule used for interferometry has a mass of ~ 10000 atomic mass unit [19], which corresponds to the mass of about 75 ^{133}Cs atoms. These interferometers are based on the Talbot effect, and are not suited for rotation sensing. Furthermore, for interferometry with much larger particles it would be necessary to make use of gratings with spacings too small to be realized with existing technologies. Aditionally, effects such as van der Waals interaction would become dominant for such gratings. Here, we propose an experiment that would reveal evidence of matter wave interference where a collection of N non-interacting, unentangled atoms (such as ^{87}Rb or ^{133}Cs) acts as a *single* particle, where N can be as large as a million. For ^{87}Rb, the corresponding Compton frequency will be $\sim 10^{31}$ (ten nonillion) Hz, and the de Broglie wavelength will be $\sim 10^{-17}$ meter (ten attometer). Furthermore, it can improve the phase measurement sensitivity by a factor as much as 10. At the same time, this type of matter wave interferometry may open up new opportunities for sensitive measurement of gravitational redshift [9] or matter wave clocks [5]. It may also serve as a testbed for macroscopic quantum decoherence due to gravitational redshift [20]

Consider an assembly of N identical independent atoms, simultaneously subjected to the $\pi/2$-dark-π-dark-$\pi/2$ sequence. If we imagine a situation where the ground state of the atoms, $|E_0\rangle \equiv |g_1, g_2, \ldots, g_N\rangle$ is coupled, directly and only, to the state where all the atoms in the ensemble were in the excited state, $|E_N\rangle \equiv |e_1, e_2, \ldots, e_N\rangle$, the resulting ensemble interferometer would experience a phase difference, $\Delta\phi_{EI} = N\Delta\phi$. However, existing technology does not enable such an excitation. Even if one were to use a pure Fock state of $N' > N$ photons, the ensemble would evolve into a superposition of $(N+1)$ symmetric collective states $|E_n\rangle\,|N'-n\rangle$, where $|N'-n\rangle$ is a state of the field with $(N'-n)$ photons, and $|E_n\rangle = J(N,n)^{-1/2}\sum_{k=1}^{J(N,n)} P_k\,|g^{\otimes(N-n)}e^{\otimes n}\rangle$, where $J(N,n) \equiv \binom{N}{n}$, P_k is the permutation operator, and $n = 0, 1, 2, \ldots, N$ [21]. Since a laser is a superposition of many Fock states, the evolution of this system under laser excitation would produce a seemingly intractable superposition of these collective states. Modeling the laser field as a semi-classical one also does not simplify the picture much [26–29]. However, we show here that, by measuring the quantum state of a single collective state, it is possible to determine the effect of the interference among all the collective states, and describe how such a measurement can be done. Choosing this collective state to be one of the two extremal states (i.e., $|E_0\rangle$or $|E_N\rangle$) also makes it possible to calculate this signal rather easily, due to the fact that the quantum state of the whole system can always be described as the tensor product of individual atomic states, assuming no interaction between these atoms. In particular, we show that for this signal, the fringe width is reduced by a factor of $\sqrt{N}$, without making use of entanglement. For the current state of the art of trapped atoms, the value of N can easily exceed 10^6, so that a reduction of fringe width by a factor of more than 10^3 is feasible. We also show that the phase fluctuation of the CSAI can be significantly smaller, by as much as a factor of 10, than that for a conventional interferometer employing the same transition and same atomic flux. The extremely narrow resonances produced in the CSAI may also help advance the field of spin squeezing [22–25], which in turn is useful for approaching the Heisenberg limit in precision metrology.

CSAI description: We choose an ensemble of N independent and non-interacting atoms of the kind described above [26]. The ensemble is prepared such that initially the i-th atom is in its ground state, $|g_i\rangle$. The ensemble is assumed to be initially situated at $(x = 0, z = 0)$ and traveling along the $\mathbf{x}$-direction with a velocity V. The ensemble undergoes the same $\pi/2$-dark-π-dark-$\pi/2$ sequence as described for the CRAI. Assuming resonant excitation, the Hamiltonian of the i-th atom after the dipole approximation, rotating-wave approximation, and rotating-wave transformation can be written as $H_i = \Omega_i\,|g_i\rangle\,\langle e_i|\,/2 + c.c.$ [27], where Ω_i is the Rabi frequency of the i-th atom. Here, a phase transformation on the Hamiltonian has also been applied to render Ω_i real. Therefore, the state of the atom initially in state $|\psi_i\rangle = c_{gi}(0)\,|g_i\rangle + c_{ei}(0)\,|e_i\rangle$ at a time, t can be expressed as $|\psi_i\rangle = (c_{gi}(0)\cos(\Omega_i t/2) - ic_{ei}(0)\sin(\Omega_i t/2))\,|g_i\rangle + (-ic_{gi}(0)\sin(\Omega_i t/2) + c_{ei}(0)\cos(\Omega_i t/2))\,|e_i\rangle$. For the sake of simplicity and brevity, we consider only the case where the intensity profile of the beams are rectangular, so that $\Omega_i = \Omega$. In a real experiment, the Rabi frequencies of each atom depend on their positions relative to the Gaussian distribution of the beam intensity profile. Due to the non-zero temperature of the trapped atoms, they also experience Doppler shift arising from thermal motion. A detailed description of the effect of these inhomogeneities on the CSAI signal is presented in the accompanying Supplementary Material [15].

A $\pi/2$-pulse is applied to the ensemble at $t = 0$. The length of the $\pi/2$-pulse is such that $\Omega\tau = \pi/2$. Immediately after the $\pi/2$ pulse, each atom in the ensemble is in state $|\psi_i(\tau)\rangle = (|g_i\rangle - i\,|e_i\rangle)/\sqrt{2}$. At this point, the first dark-zone begins and lasts for a duration of T_d. By the end of this dark-zone, the component of the atom in state $|e_i\rangle$ drifts to $(x = vT_d, z = \hbar k T_d/m)$ due to an $\hbar k$ recoil received from the laser. The state $|g_i\rangle$ continues along the $\mathbf{x}$-direction. We label the trajectories taken by $|g_i\rangle$ and $|e_i\rangle$, A and B respectively. The state of an atom at $t = \tau + T_d$ can thus be written as $|\psi(\tau + T_d)\rangle = |\psi(\tau + T_d)\rangle_A + |\psi(\tau + T_d)\rangle_B$, where $|\psi(\tau + T_d)\rangle_A = |g_i\rangle/\sqrt{2}$ and $|\psi(\tau + T_d)\rangle_B = -i\,|e_i\rangle/\sqrt{2}$. At the end of this dark zone, the ensemble encounters a π-pulse which causes the state $|g_i\rangle$ to evolve into $|e_i\rangle$ and vice-versa. The ensemble state at the end of this pulse is $|\psi(3\tau+T_d)\rangle = |\psi(3\tau+T_d)\rangle_A + |\psi(3\tau+T_d)\rangle_B$, such that $|\psi(3\tau+T_d)\rangle_A = -i\,|e_i\rangle/\sqrt{2}$ and $|\psi(3\tau+T_d)\rangle_B = -|g_i\rangle/\sqrt{2}$. Following this, the atoms are left to drift free for another dark-zone of duration T_d, at the end of which the two trajectories are redirected to converge, as shown in Fig. 1(c), and $|\psi(3\tau+2T_d)\rangle = |\psi(3\tau+T_d)\rangle$. At $t = 3\tau+2T_d$, a third pulse of duration τ is applied to the atoms. If a relative phase difference of $\Delta\phi$ is introduced between the paths, the state of the atom at the end of the last $\pi/2$-pulse is $|\psi(4\tau+2T_d)\rangle = |\psi(4\tau+2T_d)\rangle_A + |\psi(4\tau+2T_d)\rangle_B$, where $|\psi(4\tau+2T_d)\rangle_A = -i(-i\exp(-i\Delta\phi)\,|g_i\rangle + |e_i\rangle)/2$ and $|\psi(4\tau+2T_d)\rangle_B = -(|g_i\rangle - i\exp(i\Delta\phi)\,|e_i\rangle)/2$. This phase difference can be applied explicitly, or can occur, for example, due to a rotation of the entire system about an axis normal to the area carved by the trajectories.

The fringe shift at the end of the $\pi/2$-dark-π-dark-$\pi/2$ is the result of the interference of the states from the two trajectories. This is characterized by measuring the probability of finding the atom in either of the two states. The signal as a measure of the amplitude of $|g\rangle$, is therefore, $S_{CRAI} = |(1+\exp(-i\Delta\phi))/2|^2 = \cos^2(\Delta\phi/2)$. At this point we remind ourselves that the state of an ensemble is given by the direct product of its constituent atoms, $|\Psi\rangle = \prod_{i=1}^{N}|\psi_i\rangle$, where $|\Psi\rangle$ is the state of the ensemble [27, 28]. The signal of the CSAI is a measurement of any of the arising collective states. We choose to measure the state $|E_0\rangle$ which is a probability of finding all the atoms of the ensemble simultaneously in $|g\rangle$. This choice of state will be explained later on when we discuss the detection system of the CSAI. The signal of the CSAI is thus the product of the signals from the constituent atoms, $S_{CSAI} = \prod_{i=1}^{N} S_{CRAI} = \cos^2 N(\Delta\phi/2)$. It is evident from Fig. 2 that the fringe linewidth as a function of $\Delta\phi$ decreases with increasing N. We define this linewidth as the full width at half maximum (FWHM) of the signal fringe, $\varrho(N) = 2\cos^{-1}(2^{-1/2N})$. We have verified that to a good approximation, $\varrho(1)/\varrho(N) \approx \sqrt{N}$.

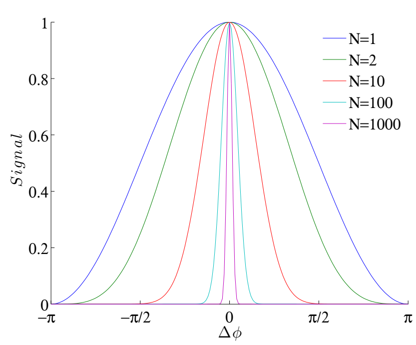

FIG. 2. Measurement of the CSAI signal (amplitude of $|G\rangle$) shows a narrowing of the fringe width such that the ratio $\varrho(1)/\varrho(N)$ increases with $\sqrt{N}$.

This narrowing can be explained by considering a composite picture of the collective excitations of the ensemble. If the ensemble in the ground state interacts with a single photon of momentum $\hbar k$, it will oscillate between $|E_0, 0\rangle \leftrightarrow |E_1, \hbar k\rangle$. Consequently, it will exhibit collective behavior such that its center of mass (COM) recoils with a velocity equal to $\hbar k/Nm$ in the direction of the absorbed photon [29]. Thus, this ensemble can be viewed as a single entity with Compton frequency N times that of a single constituent atom despite no interactions between the atoms. Conversely, the ensemble can also be pictured as a wave of $\lambda_{dB} = h/Nmv$ that is N times lower than that of a single atom, as pictured in Fig. 3. In the ideal case of uniform Rabi frequencies and Doppler shift related detunings, the first $\pi/2$-pulse splits the ensemble into a superposition of $N+1$ symmetric collective states. We have shown the corresponding interpretation of the other, more general cases in ref. 27. The state $|E_n\rangle$ receives a recoil of $n\hbar k$ due to the first $\pi/2$-pulse and is deflected in the **z**-direction by $n\hbar k T_d/Nm$ by the end of the first dark zone, making an angle $\theta_n = \tan^{-1}(n\hbar k/NmV)$ with the **x**-axis. We label the path taken by this state as Path-n. The subsequent π-pulse causes $|E_n\rangle$ to evolve to $|E_{N-n}\rangle$. This results in the deflection of the trajectory of the states so that all the $N+1$ trajectories converge by the end of the second dark-zone. The third pulse causes each of the $N+1$ states to split further. The resulting CSAI is, thus, $J(N+1, 2)$ CRAI's working simultaneously. Of these, there are x interferometers of area $(N-x+1)\Theta/N$, producing signal fringes equaling $\cos^2((N-x+1)\Delta\phi/2)$. Here x assumes values $1, 2, \ldots, N$. The interference between these cosinusoidal fringes result in the narrowing of the total fringe width. For a detailed and explicit example, please see the Supplementary Material [15].

In order to illustrate the complete picture of the proposed experiment, we consider ^{87}Rb as the atomic species as an example. We assume a scenario where the atoms will be evaporatively cooled to a temperature of about 2μK, in a dipole force trap [30] and then released. The Raman pulses wull be applied while these atoms are falling under gravity. Each Raman pulse will consist of a pair of counterpropagating, right circularly polarized ($\sigma+$) beams. One of these beams is red detuned from the $F = 1 \rightarrow F' = 1$ transition in the $D1$ manifold by $\sim$ 1.5GHz, and the other one is red detuned by the same amount from $F = 2 \rightarrow F' = 1$ transition, also in the $D1$ manifold. The second Raman beam is generated from the first one by a modulator which is driven by an ultrastable frequency synthesizer (FS) tuned to 6.8346826109GHz. We assume that the atoms are initially in the $F = 1$, $m_F = 0$ state.

Thus, the states $|g\rangle$ and $|e\rangle$ in Fig. 1(a) would correspond to the hyperfine ground states $F = 1, m_F = 0$ and $F = 2, m_F = 0$, respectively. The $\sigma+$ Raman transitions occur via the excited states $F' = 1, m_{F'} = 1$ and $F' = 2, m_{F'} = 1$. The resulting four level system can be reduced to a two level system in the same way as that for the Λ system by adiabatically eliminating the excited states together. The resulting system has a coupling rate that is the sum of the two Raman Rabi frequencies, one involving $F' = 1, m_{F'} = 1$ and the other involving $F' = 2, m_{F'} = 1$. The laser intensities at Ω_1 and Ω_2 are adjusted to ensure that the light shifts of $|g\rangle$ and $|e\rangle$ are matched.

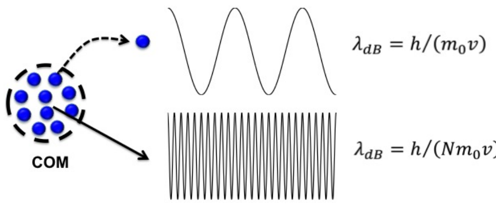

FIG. 3. λ_{dB} of an Rb-87 atom moving at a constant velocity of 300 m/s is $1.53(10^{-11}$ m). In the rest frame of the atom, its characteristic Compton frequency is $1.96(10^{25}$Hz). A cluster of 10^6 such atoms will exhibit the characteristics of a single entity of mass that is a million times that of a single Rb-87 atom. Therefore, λ_{dB} will be $1.53(10^{-17}$ m) and Compton frequency is $1.96(10^{31}$ Hz).

Detection system: At the end of the $\pi/2$-dark-π-dark-$\pi/2$ sequence, a fourth probe beam is applied to the ensemble to measure the amplitude of one of the resulting collective states, via the method of zero photon detection. In order to explain this, we revert to the three-level model of the atom and first consider a situation where the atomic ensemble is contained in a single mode cavity with volume mode V, cavity decay rate γ_c, and wavevector $k_1 = \omega_1/c$. The cavity is coupled to the atomic transition $|a\rangle \to |g\rangle$ with coupling rate $g_c = |e.\langle r\rangle|E/\hbar$, where $|e.\langle r\rangle|$ is the dipole moment of the atom and the field of the cavity is $E = \sqrt{2\hbar\omega_1/\epsilon_0 V}$. If an off-resonant classical laser pulse of frequency ω_2 is applied, the presence of the cavity will allow Raman transitions to occur between the collective states $|E_n\rangle$ and $|E_{n-1}\rangle$ with the coupling rates $\Omega'_n = \sqrt{N-n+1}\sqrt{n}\Omega'$, where $\Omega' = \Omega_2 g_c/2\Delta$. This is illustrated in Fig. 4(a).

In the bad cavity limit, where $\gamma_c \gg \sqrt{N}\Omega'$, the Raman transitions will still occur. However, the atomic system will not reabsorb the photon that has been emitted during the process, such that the transition from $|E_n\rangle$ to $|E_{n-1}\rangle$ will occur, but not vice versa. The electric field of such a photon is $E = \sqrt{2\hbar\omega_1/\epsilon_0 \mathscr{A} cT}$, where $\mathscr{A}$ is the cross-sectional area of the atomic ensemble and T the interaction time [32]. This limit applies in the CSAI where there is no cavity. In this limit, the stimulated Raman scattering is an irreversible process that can be modeled as a decay with an effective decay rate that is unique to each $|E_n\rangle$. The decay rate from $|E_N\rangle$ is $\gamma_N = 4NL|g_c\Omega_2|^2/\Delta^2 c = N\gamma_{sa}$, where $\gamma_{sa} = 16L\Omega'^2/c$ [33]. Drawing analogy from this, we conclude that the state $|E_n\rangle$ will decay at an effective rate $\gamma_n = n(N+1-n)\gamma_{sa}$.

As discussed in ref. 29, the photons under such an excitation will be emitted in the direction opposite to that of the probe beam. The experimental scheme is illustrated in Fig. 4(b). The emitted photons and the probe beam

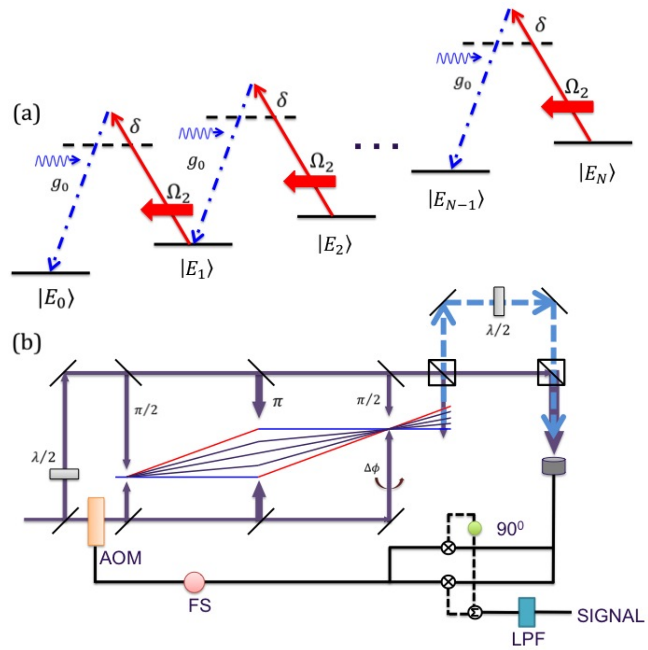

FIG. 4. (a) Atomic Interferometer experiment for an ensemble of Λ-type atoms for detecting state $|E_0\rangle$. (b) Interaction between the collective states in the bad cavity limit, which is an irreversible process.

are recombined and sent to a high speed detector, which generates a DC voltage along with a signal at the beat frequency ~ 6.834GHz with an unknown phase. This signal is bifurcated and one of the parts is multiplied by the FS signal, while the other is multiplied by the FS signal phase shifted by 90°. The signals are then squared before being combined and sent through a low pass filter (LPF) to derive the DC voltage. This DC voltage is proportional to the number of scattered photons. A lower limit is set for the voltage reading and any values recorded above it will indicate the presence of emitted photons during the probe period. The duration of the probe beam is set at $\gamma_N T = 10$, where $\gamma_N = N\gamma_{sa}$ is the slowest decay rate to ensure that even the longest lived state is allowed to decay completely. If no photon is emitted, the voltage will read below the limit, indicating that the ensemble is in state $|E_0\rangle$. If the ensemble is in any other collective state, at least one photon will be emitted. This process is repeated $\mathscr{M}$ times for a given value of $\Delta\phi$. The fraction of events where no photons are detected will correspond to the signal for this value of $\Delta\phi$. This process is then repeated for several values of $\Delta\phi$, producing the signal fringe for a CSAI.

Performance comparison: In order to compare the performance of the CSAI to that of the CRAI, we analyze the stability of the phase difference measured by them by investigating the fluctuation that has both quantum mechanical and classical components, i.e. $\delta\Delta\phi|_{total} = (\Delta S_{QM} + \Delta S_{classical})/|\partial S/\partial\Delta\phi|$, where $S(\Delta\phi)$ is the signal and $\Delta\phi$ is the phase difference introduced in the interferometer away from its center value. Since the signal depends on the phase, the fluctuations in an interferometer are not necessarily constant. Therefore, there is no unique value of signal to noise ratio (SNR) to compare unless the CSAI and the CRAI are compared at a particular value of the phase difference. Thus, the fluctuations in signal must be compared as a function of $\Delta\phi$. In ref. 15, we have given a detailed discussion of the quantum fluctuation due to quantum projection noise, $\Delta P = \sqrt{P(1-P)}$ [34], where P is the population of the state being measured, and the classical noise in the long term regime. The ratio of the phase fluctuations of the CSAI to that of the CRAI reveals that they perform comparably around $\Delta\phi = 0$ if the interferometers have perfect collection efficiency. However, as shown in ref. 15, the CRAI suffers from imperfect collection efficiency due to the latter's dependence on experimental geometry. On the other hand, the collection efficiency of the CSAI is close to unity owing to the fact that the fluorescence of photons is collected through coherent Raman scattering. As a result, for the same number of atoms detected per unit time, the CSAI is expected to outperform the CRAI by as much as a factor of 10.

In summary, we have described a CSAI with fringes as a function of $\Delta\phi$ narrowed by $\sqrt{N}$ compared to a conventional interferometer, without entanglement and violation of the uncertainty limit. We have shown that this effect is a result of the interferences among collective states, and is a manifestation of interference at a Compton frequency of ten nonillion Hz, or a de Broglie wavelength of ten attometer, for $N = 10^6$. The detection process, which is a measure of the amplitude of a collective state, can yield a net improvement of phase measurement by a significant factor.

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